LISA observations of massive black hole mergers: event rates and issues in waveform modelling

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Abstract. The observability of gravitational waves from supermassive and intermediate-mass black holes by the forecoming Laser Interferometer Space Antenna (*LISA*), and the physics we can learn from the observations, will depend on two basic factors: the event rates for massive black hole mergers occurring in the *LISA* best sensitivity window, and our theoretical knowledge of the gravitational waveforms. We first provide a concise review of the literature on *LISA* event rates for massive black hole mergers, as predicted by different formation scenarios. Then we discuss what (in our view) are the most urgent issues to address in terms of waveform modelling. For massive black hole binary inspiral these include spin precession, eccentricity, the effect of high-order Post-Newtonian terms in the amplitude and phase, and an accurate prediction of the transition from inspiral to plunge. For black hole ringdown, numerical relativity will ultimately be required to determine the relative quasinormal mode excitation, and to reduce the dimensionality of the template space in matched filtering.

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The Laser Interferometer Space Antenna (*LISA*) is being designed to detect gravitational waves of frequency between 10^{-5} and 10^{-1} Hz, with maximum sensitivity around $\sim 10^{-2}$ Hz. Astrophysical sources in this frequency band are usually split into three broad classes: 1) a large background of galactic binaries (mostly white dwarf binaries) with periods ranging from hours to tens of seconds; 2) the "extreme mass ratio inspirals" (EMRIs) of stars and black holes (BHs) of mass $M \sim 10 M_\odot$ into supermassive black holes (SMBHs); 3) the coalescence of SMBH binaries and the capture of intermediate-mass black holes (IMBHs) by SMBHs.

The distinction between IMBHs (with mass $M \sim 10^2 - 10^4\ M_\odot$) and SMBHs (with $M \sim 10^5 - 10^9\ M_\odot$) is not sharp, and we will refer to both of them collectively as "massive black holes" (MBHs). MBHs will be observed with large signal-to-noise ratio (SNR), allowing precise measurements of the source parameters and tests of the strong-field effects of general relativity, both in the inspiral [1, 2] and ringdown [3, 4] phases. For this reason the observation of MBH mergers is one of the top *LISA* science milestones.

The data analysis strategy to observe MBH mergers will be affected by two key factors: the event rate of mergers emitting gravitational radiation at frequencies in the *LISA* best sensitivity window, and the accuracy of our knowledge of the gravitational waveforms. In Sec. 1 we review present estimates of the event rates for both SMBH binaries and binaries comprising one IMBH. In Sec. 2 we provide a list of important open problems in our knowledge of the theoretical waveforms. For the inspiral phase these include spin precession, eccentricity, the inclusion of high-order Post-Newtonian terms in the amplitude and phase, and an accurate prediction of the transition from inspiral to plunge. For the ringdown phase we point out that numerical relativity will ultimately be required to determine the relative quasinormal mode excitation, and to reduce the dimensionality of the template space in matched filtering.

1. Massive black hole binary event rates

The present observational evidence for the existence of astrophysical BHs is strong and growing [5]. The most convincing case comes from observations of stellar proper motion in the center of our own galaxy, indicating the presence of a "dark object" of mass $M \simeq (3.7 \pm 0.2) \times 10^6\ M_\odot$ [6] and size smaller than about one astronomical unit [7]. A Schwarzschild BH of the given mass has radius $R = 2GM/c^2 \simeq 0.073$ astronomical units, compatible with the observations. Any distribution of individual objects within such a small region would be gravitationally unstable [8], and theoretical candidates alternative to BHs (such as boson stars and gravastars) are probably unlikely to exist in nature. Furthermore there is strong observational evidence for the presence of MBHs in the bulges of nearly all local, massive galaxies [9]. These BHs have masses in the range $M \sim 10^5 - 10^9\ M_\odot$, approximately proportional to the mass of the host galaxies, $M \sim 10^{-3}\ M_{\rm galaxy}$ [10]. Recent observations led to other remarkable discoveries. There is an almost-linear relation between the mass of a MBH and the mass of the galactic bulge hosting the BH [9]. The BH mass is also tightly correlated with other properties of the galactic bulge, such as the central stellar velocity dispersion σ, the bulge light concentration and the near-infrared bulge luminosity [11]. Applying over an enormous mass range, these correlations clearly indicate that MBHs are somehow aware of the surrounding galactic environment.

Details of the formation process of MBHs are not well known. A popular formation scenario involves the collapse of primordial, massive ($M \sim 30 - 300\ M_\odot$)

metal free Population III stars [12] at cosmological redshift $z \sim 20$ to form primordial BHs with $M \sim 10^2\ M_\odot$, clustering in the cores of massive dark-matter halos [13]. In some alternative scenarios, BH seeds of larger mass $M \sim 10^5\ M_\odot$ form at $z \gtrsim 12$ from low-angular momentum material in protogalactic discs [14] (see also [15]). A major source of uncertainty in predicting the evolution of MBHs comes from the unknown "occupation number" (fraction of galaxies containing a MBH) at high redshifts. As dark matter halos merge (maybe starting early, at $z \sim 20$), seed MBHs can grow both through gas accretion (which is perhaps the dominant mechanism [16]) and through coalescence with other MBHs.

Larger galaxies grow through the agglomeration of smaller galaxies and protogalactic fragments. If more than one of these fragments contains a MBH, MBHs will form a bound system in the merger product [17]. The electromagnetic observation of a MBH binary is a hard task, requiring the study of some emission component close to the BHs. So far there is no completely convincing observational case for the detection of "hard" MBH binaries, that is, binaries having orbital velocities larger than the velocity dispersion of stars in the galactic nucleus [18].

The formation of MBHs during galaxy mergers is a challenging problem in theoretical astrophysics. The general scenario was outlined in the pioneering analysis of [17], and an excellent review of the state of the art in this field can be found in [18]. The evolution of a MBH binary can be roughly divided in three phases: i) as the galaxies merge, MBHs sink to the center via dynamical friction; ii) the binary's binding energy increases because of gravitational slingshot interactions: the ejection of stars on orbits intersecting the binary (these stars' angular momentum must be in a region of phase space called the "loss cone"); iii) if the binary separation becomes small enough, gravitational radiation carries away the remaining angular momentum. Notice that the gravitational wave coalescence time is shorter for more eccentric binaries [19], and as a result high-eccentricity binaries could be more likely to coalesce within a Hubble time. In Sec. 2 we will briefly sketch the complications in data analysis (and the advantages in terms of parameter estimation) for binaries with non-zero eccentricity.

The transition from phase ii) to phase iii) is a field of active research, that has been referred to as the "final parsec problem" [18]. Since the binary will quickly eject all stars through gravitational slingshot interaction, the problem is to find some mechanism to refill the loss cone. Unfortunately N-body simulations of spherical galaxies do not provide very reliable answers. The reason is that the binary's hardening rates depend strongly on N, roughly decreasing as N^{-1} at the largest values of N allowed by present simulations. Proposed mechanisms to overcome the final parsec problem include gas accretion, star-star encounters and triaxial distortions of galactic nuclei [20]. Recent simulations show that if the galaxy is allowed to rotate, hardening rates become independent of N and binaries *do* coalesce within a Hubble time [21].

These recent results are consistent with some observational evidence indicating that *efficient coalescence is the norm.* First of all, as we mentioned earlier, at present there is no convincing evidence for bound SMBH binaries. In galaxies containing an uncoalesced binary, mergers would bring a third black hole into the nucleus, and the resulting gravitational slingshot interaction would eject one or more MBHs from the nucleus. This would produce off-center MBHs, but so far off-center MBHs have escaped detection. Since we don't detect off-center black holes‖, coalescence must

‖ In [22] three-body slingshot interactions were proposed to explain the observations of the bright quasar HE0450-2958, which seems not to be surrounded by a massive host galaxy. See however [23] for an alternative explanation that does not require the quasar to be ejected.

proceed on short timescales. In addition, Haenhelt [24] remarked that MBH ejections by three-body slingshot interactions would weaken the tight $M - \sigma$ correlations that are observed. More speculative evidence for efficient coalescence comes from observations of radio galaxies. About a dozen radio galaxies exhibit abrupt changes in the orientation of their radio lobes, producing an X-shaped morphology. According to some theoretical models, the MBH producing the jet could have undergone a spin flip, possibly produced by capture of a second MBH: perhaps in X-shaped radio sources we are *already* observing merger events [25].

The third (gravitational-radiation dominated) phase in the evolution of MBH binaries has recently attracted a lot of attention from the astrophysics community. The reason is that, according to general relativity, unequal-mass binaries should radiate not only energy and angular momentum, but also linear momentum [26]. The resulting gravitational wave recoil speed could be large enough to kick MBHs out of the host galaxy [27]. Unfortunately, a large fraction of this linear momentum would be radiated in the final phases of the MBH binary coalescence, where black hole perturbation theory and Post-Newtonian (PN) expansions of the Einstein field equations are less reliable. Recent PN calculations set an upper limit of $v_{\rm kick} \sim 250 \pm 50$ km s^{-1} on the resulting recoil speed [28]. Observationally, MBH ejections by gravitational wave recoil would produce some scattering in the $M-\sigma$ relation. Recent arguments suggest that the observed scattering already constrains the magnitude of the kick to values $v_{\rm kick} \lesssim 500$ km s^{-1}, compatible with the general relativistic upper limit [29]. In the near future further observations may rule out gravitational wave recoil as a viable mechanism for MBH ejection from galactic cores. Calculations assuming the largest possible value allowed for the magnitude of the kick show that, if viable, gravitational wave recoil would lower dynamical friction, hence lower the rates of MBH binaries observable by *LISA* by factors ~ 10 [30].

The *LISA* noise curve determines the optimal mass and redshift range where binary inspiral and ringdown events have large SNR, allowing a precise measurement of the source parameters (see Fig. 1 below). Reliable estimates of the number of events detectable during the mission's lifetime, and of their mass spectrum as a function of redshift, will play a key role in the planning of *LISA* data analysis. For this reason, over the last few years the calculation of MBH merger event rates and of their mass spectrum has become an active field of research.

In a pioneering paper [31] Haehnelt noticed that the event rate inferred from the quasar luminosity function is too low to be detectable, but event rates can be very high if we assume that MBHs reside in dark matter halos. Various authors have recently re-computed these event rates. A major factor influencing their predictions is the model used to deal with the merger history of dark halos. Menou *et al.* use merger tree algorithms to show that the ubiquity of MBHs in luminous galaxies today [9] is consistent with a small "occupation number" (fraction of galaxies containing a MBH) at high redshifts. They predict an integrated rate of ~ 10 events/year for MBH mergers out to $z \sim 5$ [32]. Wyithe and Loeb use a semianalytic model of dark matter halo mergers which assumes that all halos contain MBHs, thus overestimating the event rate by about one order of magnitude [33]. Revised estimates by Rhook and Wyithe using a more conservative occupation number yield rates of 15 events/year, consistent with [32], and suggest that most events should originate at $z \sim 3-4$ [34]. The estimates in [32, 34] should be considered somewhat optimistic, in the sense that both works assume coalescence to be rapid. Sesana *et al.* use a more conservative approach to coalescence: in their model for binary evolution some binaries can be

ejected from the galactic core. Using a seeded MBH growth model, they estimate that 3 years of *LISA* observations could resolve ~ 35 MBH mergers in the redshift range $2 \lesssim z \lesssim 6$. A fraction of these mergers (~ 9 events/year) would contain at least one black hole heavier than $10^5\ M_\odot$ [35, 36]. Enoki *et al.* use a semianalytic model of galaxy and quasar formation based on the hierarchical clustering scenario to estimate the stochastic background due to inspiralling MBH binaries. They find that *LISA* could detect binaries with total mass $M < 10^7\ M_\odot$ and $z > 2$ at a rate of 1 event/year; events with $M \sim 10^8$ would mostly be observed at $z < 1$, and events with $M \sim 10^6$ would be visible at $z \sim 3$ (though with lower amplitude) [37].

Other authors predict more optimistic event rates than those listed so far. Islam *et al.* estimate that, if merger is efficient, $10^4 - 10^5$ events/year could be observed in the *LISA* band; these events could also be coincident with gamma-ray burst observations [38]. Scenarios in which BH seeds of large mass $M \sim$ a few $\times\, 10^5\ M_\odot$ form at $z \gtrsim 12$ from low-angular momentum material in protogalactic discs [14] predict even larger rates. In fact, in these scenarios MBHs should produce a noisy stochastic background similar to the white dwarf binary background, but with much larger SNR [39].

Table 1. SMBH binary rates (events/year) predicted by different models.

Reference	Rate	Redshift range
Haenhelt 2003 [24]	0.1-1	$0 < z < 5$ (gas collapse only)
	10-100	$z > 5$ (hierarchical buildup)
Menou *et al.* 2001 [32]	10	$z < 5$
Rhook and Wyithe 2005 [34, 33]	15	$z \sim 3 - 4$
Sesana *et al.* 2004 [35, 36]	35	$2 < z < 6$
	9	one BH with $M > 10^5 M_\odot$
Enoki *et al.* 2004 [37]	1	$z > 2$
Islam *et al.* 2003 [38]	$10^4 - 10^5$	$z \sim 4 - 6$
Koushiappas and Zentner 2005 [39]	stochastic background	mostly $z \sim 10$, down to $z \sim 1$ (see their Fig. 3)

Given the significant differences between MBH binary formation models and the predicted event rates, we find it useful to provide a schematic summary of the available literature on event rates in Table 1. The numbers we list should be interpreted with caution. Each prediction depends on a large number of poorly known physical processes, and the notion of "detectability" of a merger event is defined in different ways: some authors define detectability setting a threshold on the SNR, others set a threshold on the gravitational wave effective amplitude. Furthermore, different authors use different *LISA* noise curves. Some of them assume that the low-frequency f^{-2} dependence of the *LISA* acceleration noise can be extrapolated below 10^{-4} Hz, which increases the event rates by including highly redshifted, high mass BH mergers. In reality the noise curve will probably rise very steeply below $\sim 3 \times 10^{-5}$ Hz. This affects also the SNR and parameter estimation (see [1] for a discussion).

A tentative bottom line is that we could face one of the following two scenarios. According to a class of models, we should observe ≈ 10 events/year at redshifts (say) $2 \lesssim z \lesssim 6$. However, we cannot exclude the possibility that hundreds or thousands of MBHs will produce a large (and perhaps stochastic) background in the *LISA* data. Clearly, the detection strategy to use strongly depends on which of the two scenarios

actually occurs in nature. At this stage, our best bet is to devise techniques of detection and parameter estimation that encompass both possibilities.

1.1. Binaries involving intermediate-mass black holes

Until recently astrophysical BHs were thought to belong to either one of two broad mass ranges: stellar-mass BHs with $M \sim 3 - 20\ M_\odot$ (produced by the collapse of massive stars) and SMBHs with $M \sim 10^5 - 10^9\ M_\odot$, which have been the main focus of our discussion so far. In the last few years observations of ultraluminous X-ray sources, combined with the fact that several globular clusters show evidence for an excess of dark matter in their cores, provided strong hints of the existence of astrophysical IMBHs with $M \sim 10^2 - 10^4\ M_\odot$ (see [40] for a review).

It is possible that MBH binaries have a mass ratio $q \equiv m_2/m_1$ significantly smaller than one. In fact, recent studies by Volonteri *et al.* [16, 41] suggest that low-redshift ($z \lesssim 10$) MBH mergers predominantly occur with a mass ratio $q \simeq 0.1$ or smaller. Some of these binaries could be SMBH-IMBH binaries. If MBH binary coalescence timescales are long enough, three-body slingshot interactions and gravitational wave recoil may generate a population of IMBHs wandering in galaxy halos at the present epoch [42].

Event rates for IMBH-IMBH binaries (that is, binaries containing a $10 - 100\ M_\odot$ BH orbiting a $100 - 1000\ M_\odot$ BH) were first estimated by Miller [43] and then revised by Will [44]. The revised estimates are very pessimistic, predicting $\sim 10^{-6}$ events/year for typical values of the parameters.

A more promising scenario for gravitational wave detection emerges if IMBHs play a role in the formation of SMBHs. The process can roughly be split in three stages [45]. In the first stage IMBHs are formed by runaway mergers of massive stars in dense young stellar clusters [45], or alternatively by runaway mergers of smaller black holes in globular clusters [46], which are typically much older (so that all stars of mass $\gtrsim 0.8\ M_\odot$ have evolved off the main sequence, and the most massive objects are compact remnants) ¶. In the second stage these IMBHs accumulate at the galactic center due to sinkage of the clusters by dynamical friction. In the third and final stage, the IMBHs merge (either by successive multi-body interactions or spiralling into a preexisting central SMBH) emitting gravitational radiation.

Matsubayashi *et al.* [47] estimated the inspiral and ringdown radiation emitted in the formation of a $10^6\ M_\odot$ SMBH by merging of a thousand IMBHs of mass $10^3\ M_\odot$, following two radically different merging histories. In the *hierarchical* scenario pairs of equal-mass BHs merge producing a single, more massive BH, and then the process repeats. Conversely, in the *monopolistic* (or runaway) scenario, a single BH grows through subsequent mergers with surrounding BHs. In the hierarchical scenario the majority of mergers occurs between low-mass black holes, so most of the radiation is emitted in the *LISA* high frequency band (or even at frequencies $f \gtrsim 10$ Hz, too large to be detected by *LISA*). In the monopolistic scenario, many events occur when the mass of the accreting black hole is large enough for the radiation to be in the optimal *LISA* band. Event rates are extremely uncertain and depend in a similar way on the merging history. In the monopolistic scenario they could be as high as $20 - 70$ events/year if all galaxies experience mergers of $\sim 10^3\ M_\odot$ BHs. In the hierarchical

¶ The viability of runaway mergers in young stellar clusters is supported by numerical simulations, while in the globular cluster scenario growth times are rather long, making it harder to explain why young clusters (such as MGG-11) would contain IMBHs.

scenario, given the poor sensitivity of *LISA* at high frequencies, they could be smaller by about one order of magnitude[+].

A detailed N-body simulation of the sinking of a 3000 $M_\odot$ IMBH into a 6×10^6 $M\odot$ SMBH can be found in [49]. Dynamical friction becomes ineffective at the orbital radius inside which the initial stellar mass is comparable with the IMBH mass. Quite surprisingly, simulations show that at this point the IMBH's orbital eccentricity grows, lowering the merging timescale due to gravitational radiation. Variants of this scenario have been considered by Miller and by Portegies-Zwart *et al.* [50, 51]. Miller estimated a detection rate of a few events/year, and suggested that mergers of a 10^3 $M_\odot$ IMBH into a 10^6 $M_\odot$ SMBH could be observed out to $z \sim 20$ at SNR 10 in a one-year integration. Typical SNRs could be much larger than the typical SNRs for EMRIs ($\sim 10^3$ instead of ~ 10), allowing precise parameter estimation [50] (but see below for theoretical issues in modelling the waveforms of SMBH-IMBH binaries). Portegies-Zwart *et al.* predict an even more optimistic rate of $\sim 10^2$ events/year throughout the universe [51]. These estimates are very preliminary and even more uncertain than the corresponding estimates for SMBH binaries, but they should be taken into account to decide (for example) the optimal armlength of *LISA*.

2. Open problems in waveform modelling

In the last decades inspiral waveforms for circular orbits, which are crucial for the detection of gravitational waves by ground-based interferometers, have been studied in depth. Expansions of the phasing are known up to 3.5PN order if spin terms are negligible, and up to 2PN order for binaries with spins aligned (or antialigned) and normal to the orbital plane. Leading-order contributions to the phasing from alternative theories of gravity (scalar-tensor theories and theories allowing for a non-zero graviton mass) have also been studied in the context of *LISA* (see [1, 2] for an extensive discussion).

The measurability of ringdown waves with *LISA* has been studied in [3]. For a Schwarzschild BH the real part of the fundamental quasinormal mode frequency $\omega = 2\pi f + \mathrm{i}/\tau$ is in the optimal region of the *LISA* sensitivity curve, $f = 1.207 \times 10^{-2}(10^6 M_\odot/M)$, and the damping time $\tau = 55.37(M/10^6 M_\odot)$ is slightly larger than the light travel time across one of the *LISA* arms $T_{\rm light} = (5 \times 10^9\ \mathrm{m})/c \simeq 17$ s.

The main uncertainty affecting the ringdown SNR concerns the ringdown efficiency $\epsilon_{\rm rd}$ (ratio of energy radiated to the BH mass), plausible values of which range perhaps between 0.1% and 3%. The effect of angular momentum is less pronounced [3]. In Fig. 1 we plot the SNR for observations of the last year of inspiral of equal-mass BH binaries (as a function of the binary's total mass in the source frame), and compare it with the SNR for the ringdown of the finally formed BH (as a function of its mass in the source frame). We compute this quantity for different values of the cosmological redshift, assuming a cosmology with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 0.72$. The inspiral and ringdown SNRs are comparable. For example, at $D_L = 3$ Gpc ($z \simeq 0.5$) the ringdown SNR can be as large as $\sim 10^4$ for BHs of mass $\sim 4 \times 10^6$ $M_\odot$, and the inspiral SNR can be as large as $\sim 4 \times 10^3$ for a total mass of the binary $\sim 7 \times 10^5$ $M_\odot$.

[+] Seto *et al.* [48] proposed to shorten the *LISA* armlength by a factor $\sim$ 10. The best sensitivity region of the resulting instrument (*DECIGO*) would be shifted towards higher frequencies, dramatically improving the event rates in the hierarchical scenario. The price to pay is that *DECIGO* would have low sensitivity to gravitational waves from BH binaries of mass $\gtrsim 10^5$ $M_\odot$.

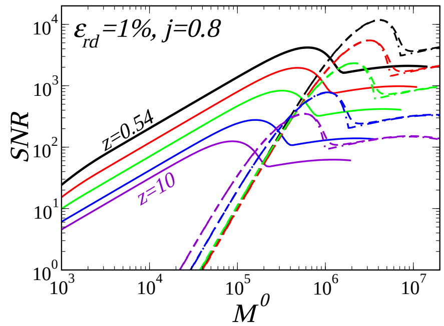

Figure 1. SNR for the last year of inspiral as a function of the total mass of the binary (continuous lines) and SNR for ringdown as a function of the BH mass (dashed lines). In both cases the masses M^0 are evaluated in the source frame: the mass measured at the detector is $M = (1 + z)M^0$. For the ringdown we pick the fundamental $l = m = 2$ quasinormal mode frequency with specific angular momentum $j = 0.8$, assuming an efficiency $\epsilon_{\rm rd} = 1\%$. From top to bottom the lines correspond to redshifts $z = 0.54$, $z = 1$, $z = 2$, $z = 5$ and $z = 10$ (from [3]).

Since the SNR is so large and errors on the source parameters scale with the inverse of the SNR, *LISA* can provide very accurate measurements of the source parameters both for inspiral [1, 2] and for ringdown [3]. This suggests the exciting possibility to measure (say) the masses of the BHs in a binary, or the mass of the BH they form after merger, by matched filtering of the inspiral and ringdown gravitational waveforms, respectively. A problem with this idea is that *LISA* does not measure masses in the source frame M^0, but only a redshifted combination $M = (1 + z)M^0$. A possibility to disentangle the z-dependence is to measure the luminosity distance $D_L(z, \Omega_M, \Omega_\Lambda, H_0)$ and (assuming that cosmological parameters are known to a good accuracy) invert this relation to get $z(D_L, \Omega_M, \Omega_\Lambda, H_0)$ [52]. In the range where we expect most events (say $2 \lesssim z \lesssim 6$) the error on D_L is rather small, typically less than $\sim 10\%$: see eg. Fig. 7 in [1]. By the time *LISA* flies, weak lensing errors can be expected to dominate over other sources of error [53, 54]. If SMBH mergers are accompanied by gas accretion leading to Eddington-limited quasar activity, and if spin precession reduces the errors in parameter estimation, the *LISA* error volume may be small enough to contain a single quasar out to $z \sim 3$, allowing a test of the hypothesis that gravitational wave events are accompanied by bright quasar activity [54].

This ambitious program relies on a detailed knowledge of the gravitational waveforms, which is necessary to reduce errors in parameter estimation. Our present knowledge of theoretical templates should be extended to include the following effects:

1) Spin precession - In [1] we observed that including spin-orbit and spin-spin terms in the gravitational wave phasing degrades the accuracy of parameter estimation. This is easy to understand, since the spin-orbit and spin-spin parameters are strongly correlated with other parameters in the phasing (such as the masses) and adding more parameters effectively dilutes the available information. However in [1] we considered only spins aligned (or antialigned) and normal to the orbital plane, which is probably not realistic. In general the relativistic spin-orbit interaction causes the orbital plane to precess in space, producing a characteristic modulation of

the waveforms. A preliminary analysis by Vecchio shows that, for a $(10^6 + 10^6)$ $M_\odot$ binary, the additional information coming from spin precession can improve parameter estimation by factors $\sim 3 - 10$ in angular resolution and luminosity distance, and by factors ~ 10 and $\sim 10^3$ in chirp mass and reduced mass, respectively [55]. A more general and systematic analysis is urgently needed.

2) Eccentricity - For Earth-based interferometers, neglecting the orbital eccentricity of a binary is an excellent approximation. Earth-based detectors can only observe binaries in the very last stages of the inspiral, when radiation reaction has had enough time to circularize the orbits [19]. For *LISA* the situation is different. As we discuss below, MBH binaries could have a significant eccentricity in the last year of inspiral, especially if their mass ratio is not close to one (which is the case for SMBH-IMBH binaries). It is well known that orbital eccentricity produces radiation at all harmonics of the Keplerian frequency f_K: $f_{GW} = nf_k$ $(n = 1,\ 2,\ 3\ldots)$ [19]. For high-mass SMBH binaries the gravitational wave frequency for circular orbits $f_{GW} = 2f_K$ could be too low to be detected by *LISA* with high SNR, but higher harmonics may be in the optimal frequency band. In addition, the presence of higher harmonics in the signal effectively provides more information on the source, improving parameter estimation [56].

Analytical calculations and N-body simulations show that, in purely collisionless spherical backgrounds, the expected equilibrium distribution of eccentricities is skewed towards high $e \simeq 0.6 - 0.7$, and that dynamical friction does not play a major role in modifying such a distribution ([57], in particular Fig. 5). Recent simulations using smoothed particle hydrodynamics follow the dynamics of binary BHs orbiting in massive, rotationally supported circumnuclear discs [58]. The rotation of the disc circularizes the orbit if the binary *corotates* with the disc, possibly increasing the merging timescale due to gravitational radiation so much that the binary stalls and no coalescence results. If the orbit is *counterrotating* the initial eccentricity does not decrease, and BHs may enter the gravitational wave emission phase with high eccentricity. For corotating discs, the numerical resolution of these simulations is not good enough to compute the precise value of the (small) residual eccentricity when the BHs are close enough that gravitational radiation takes over.

Complementary studies show that eccentricity evolution may still occur in later stages of the binary's life, because of close encounters with single stars [59] and/or gas-dynamical processes [60]. Stellar dynamical hardening might leave the binary with non-zero eccentricity, although most studies suggest that any such eccentricity would be small [59] (see however [49] and [61] for recent examples of eccentricity growth in N-body simulations). On the other hand, the gravitational interaction of the binary with a surrounding gas disc is likely to increase the eccentricity of BH binaries. The transition between disc-driven and gravitational wave-driven inspiral can occur at small enough radii that a small but significant eccentricity survives, typical values being $e \sim 0.02$ (with a lower limit $e \simeq 0.01$) one year prior to merger (cf. Fig. 5 of [60]). If the binary has an "extreme" mass ratio $q \lesssim 0.02$ the residual eccentricity can be considerably larger, $e \gtrsim 0.1$.

For concreteness, in Fig. 2 we show the SNR for different harmonics of an eccentric binary at luminosity distance $D_L = 3$ Gpc, observed during the last year of inspiral. We assume that the eccentricity $e = 0.33$ one year prior to merger. The more massive BH has $m_1 = 10^6$ $M_\odot$, and we plot the SNR of different harmonics as a function of the lighter BH's mass m_2 (in $M_\odot$). Harmonics with $n = 2$ to $n = 5$ are detectable for all values of the mass ratio. For low mass ratios $q \lesssim 0.01$ (that is, for SMBH-IMBH binaries

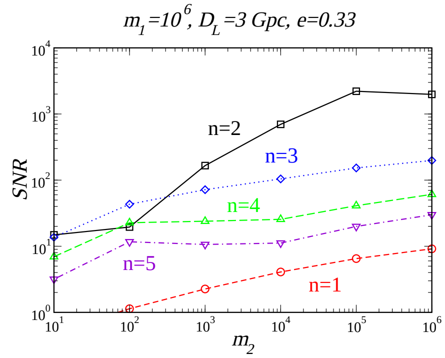

Figure 2. *(Courtesy of Jim Shifflett)* SNR for different harmonics of an eccentric binary at luminosity distance $D_L = 3$ Gpc, observed during the last year of inspiral. We assume that the eccentricity $e = 0.33$ one year prior to merger. The more massive BH has $m_1 = 10^6\ M_\odot$, and we plot the SNR of different harmonics as a function of the lighter BH's mass m_2 (in $M_\odot$).

and EMRIs) the SNR of the $n = 3$ harmonic can be comparable with the SNR of the $n = 2$ harmonic. A more extensive survey of the parameter space is clearly needed [62]. For SMBH-IMBH binaries, including eccentricity could be necessary for detection. For SMBH binaries it will be very useful for detection of mergers in the high-mass end ($M \gtrsim 10^7\ M_\odot$), and possibly important for parameter estimation.

3) High-order PN effects in phasing and amplitude - The *LISA* SNR for MBH inspirals can be very large (see Fig. 1). When the SNR is so large we must take into account the possibility that *systematic* errors, due to the truncation of the phasing at some given PN order, could be comparable with *statistical* errors. Preliminary results [63] show that the contribution of high-order PN terms in the phasing is not particularly significant for equal-mass mergers, but can be very relevant when the mass ratio is small. As a simple measure of the convergence properties of the PN expansion we can compute the number of cycles from different PN contributions. For a $(10^6 + 10^6)\ M_\odot$ binary, the Newtonian, 1PN, 2PN and 3PN terms contribute $(\sim 2300, \sim 100, \sim 5, \sim 2)$ cycles, respectively. As the mass ratio decreases the PN expansion gets worse. For a $(10^6 + 10^5)\ M_\odot$ binary the relative contributions are $(\sim 5000, \sim 300, \sim 10, \sim 2)$ cycles, and for a SMBH-IMBH binary of $(10^6 + 10^3)\ M_\odot$ they become $(\sim 27000, \sim 4000, \sim 400, \sim 30)$ [63].

The bottom line is that for SMBH-IMBH binaries, not only high-order harmonics have large relative SNR if $e \neq 0$: high-order PN terms contribute many cycles even for zero eccentricity. Since the PN approximation becomes inaccurate for these systems, one could think about using the Teukolsky formalism (based, roughly speaking, on an expansion in q keeping only the linear term). However for SMBH-IMBH binaries the mass ratio can be rather large ($q \gtrsim 10^{-3}$), and the accuracy of the Teukolsky formalism becomes questionable. This "buffer zone" between EMRIs and SMBH binaries calls for the development of a hybrid approach, taking the best from both the PN expansion and the Teukolsky formalism.

Preliminary results show that the usual "restricted PN approximation" (where the amplitude is computed using the quadrupole formula, and only PN corrections in the phasing are considered) may be inappropriate for *LISA*. The inclusion of leading-

order PN corrections to the *amplitude* may be necessary for detection, even for SMBH binaries with moderate mass ratios $q \sim 0.1$ [64].

4) Transition from inspiral to plunge - For ground-based interferometric detectors, SNR estimates suggest that the first detection may concern BH binaries of total mass $M \gtrsim 25\ M_\odot$. In this case the most useful part of the waveform is emitted in the last ~ 5 orbits of the inspiral and during the plunge, that takes place after crossing the last stable orbit [65]. The transition from inspiral to plunge is not so important for MBH observations with *LISA*, since typically we should be able to observe thousands of cycles of inspiral. However, if the *LISA* acceleration noise is not under control below (say) $\sim 10^{-4}$ Hz, knowledge of the transition from inspiral to plunge could be important to detect high-mass ($M \gtrsim 10^7\ M_\odot$) binaries. An accurate model of this phase would also be useful to predict the initial conditions of the merger, eventually leading to an estimate of the relative quasinormal mode excitation in the ringdown [4]. Most importantly, the transition from inspiral to plunge is crucial to estimate the kick velocity. The reason is that (as pointed out in [28]) the maximum velocity accumulated in the inspiral phase is ~ 20 km s^{-1}, so that the largest contribution to the kick comes from the plunge.

5) Merger and ringdown waveforms - Ringdown waveforms are linear superpositions of damped exponentials whose frequencies f and damping times τ (or equivalently, quality factors $Q = \pi f \tau$) are well known. The main uncertainty here comes from our poor knowledge of the merger phase in generic situations (black holes with different masses, spins, spin orientations etcetera), which in turn affects our knowledge of the energy distribution between different modes (see [3] and [4] for a detailed discussion). Numerical relativity will ultimately tell us which angular components (more technically: which values of (l, m) in the spin-weighted spheroidal harmonic decomposition of the perturbations) are excited in a realistic merger. This is an important issue, since it will determine whether we can use *LISA* to test the no-hair theorem through observations of the ringdown [3].

Creighton [67] provides a simple estimate of the number of filters needed for detection of (single-mode) ringdown signals for ground-based detectors. His estimate carries over directly to the case of *LISA*. Assuming that the noise power spectrum is approximately constant over the frequency band of two neighbouring filters, and that our template bank in the $(f,\ Q)$ plane covers the range $0 \le Q \le Q_{\rm max}$, $f_{\rm min} \le f \le f_{\rm max}$, the minimum number of filters we need is

$$N_{\rm filters} \sim \frac{1}{4\sqrt{2}} (ds^2_{\rm max})^{-1} Q_{\rm max} \ln\left(\frac{f_{\rm max}}{f_{\rm min}}\right) \simeq 1085 \left(\frac{Q_{\rm max}}{20}\right)\left(\frac{ds^2_{\rm max}}{0.03}\right)^{-1}$$
$$\times \left\{1 + \frac{1}{\log 10^4}\left[\log\left(\frac{f_{\rm max}}{1\ {\rm Hz}}\right) - \log\left(\frac{f_{\rm min}}{10^{-4}\ {\rm Hz}}\right)\right]\right\} .$$

Here ds^2 is a metric measuring distances in the template space [68], and a maximum distance $ds^2_{\rm max} = 3\%$ corresponds to losing 10% of the expected event rate due to a mismatched template. For detection of N single-mode waveforms, an estimate of the filters we need can be obtained multiplying the previous number by N. The problem of determining the number of filters required to *resolve* two or more modes [3] has not been discussed so far. Clearly, knowing *which* modes are excited in a realistic merger can dramatically reduce the dimensionality of the template space.

3. Conclusions and outlook

The observation of MBH mergers is potentially the most rewarding *LISA* milestone. The data analysis strategy will depend on two key elements: the event rate of mergers emitting gravitational radiation at frequencies in the *LISA* best sensitivity window, and the accuracy of our knowledge of the gravitational waveforms.

In Table 1 we provide a schematic summary of event rate estimates. As testified by the spread between different models, these estimates depend on a large number of poorly known physical processes. In making comparisons we must also account for the fact that different authors use different notions of "detectability" of a merger event and different models of the *LISA* noise curve (event rates are particularly sensitive to the assumed *LISA* acceleration noise below 10^{-4} Hz). Roughly speaking, we could face one of the following two scenarios. According to a class of models, we should observe ≈ 10 events/year at redshifts $2 \lesssim z \lesssim 6$. In the second class of models, hundreds or thousands of MBHs will produce a large (and perhaps stochastic) background in the *LISA* data: to borrow the terminology used for the white dwarf binary background, this would be a very noisy "cocktail party problem". We should be ready for any of the two scenarios to actually occur in nature, devising techniques of detection and parameter estimation that encompass both possibilities.

The physics we can learn from the observations will depend on our ability to develop reliable theoretical templates for the waveforms. For the inspiral phase, the most important effect should be the strong modulation in the waveforms produced by spin precession. An important difference with Earth-based detectors is that (depending on the binary's formation process) the inspiral signal could have significant residual eccentricity when it enters the *LISA* band. Furthermore, given *LISA*'s potentially large SNR, higher-order PN effects in phasing and amplitude must be included to improve parameter estimation. Eccentricity and higher-order PN corrections are more relevant for small mass ratios (see eg. Fig. 2), and their inclusion will be necessary in the data analysis of SMBH-IMBH binaries. An accurate model of the transition from inspiral to plunge should only be necessary for high-mass binaries if the MBH mass spectrum as a function of redshift is such that a significant number of events have frequencies below 10^{-4} Hz, *and* if *LISA* design choices do not guarantee complete control of the acceleration noise in this frequency band.

SMBH-IMBH binaries are promising sources, despite their largely uncertain event rates. They present us with a different challenge, stretching the applicability limits of both PN expansions and black hole perturbation theory. A pragmatic data analysis approach for these systems could adopt hybrid approximation schemes, similar to the "kludge" waveforms for EMRIs (see S. Drasco's contribution to these proceedings).

Simulations of the merger in numerical relativity will be useful to assess the astrophysical significance of gravitational wave recoil. They will also determine which modes are dominant in the ringdown phase, reducing the dimensionality of the template space required for ringdown detection and for tests of the no-hair theorem. Reliable estimates of the energy and angular momentum emitted in "generic" merger conditions (where binary members have arbitrary mass, spin magnitude and orientation) will provide valuable information to match inspiral and ringdown waveforms, significantly improving parameter estimation from both phases.

The detection of gravitational waves from MBH mergers is a challenging interdisciplinary task, and it could open an exciting new era for astronomy. We should be ready for surprises.

Acknowledgments

I am grateful to Matt Benacquista, Alessandra Buonanno, Vitor Cardoso, Kristen Menou, Edward Porter, Massimo Tinto, Michele Vallisneri, Cliff Will and an anonymous referee for suggestions. I thank Jim Shifflett for kindly allowing me to include his preliminary results on eccentric binaries. This work was supported in part by the National Science Foundation under grant PHY 03-53180.

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